

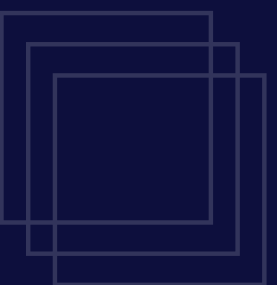


DOES THE OPTIONS MARKET **UNDERREACT** TO FIRMS' LEFT TAIL RISK?

Based on Chen, Gan & Vasquez (2024) | Journal of Financial & Quantitative Analysis

Group 2

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SCOPE

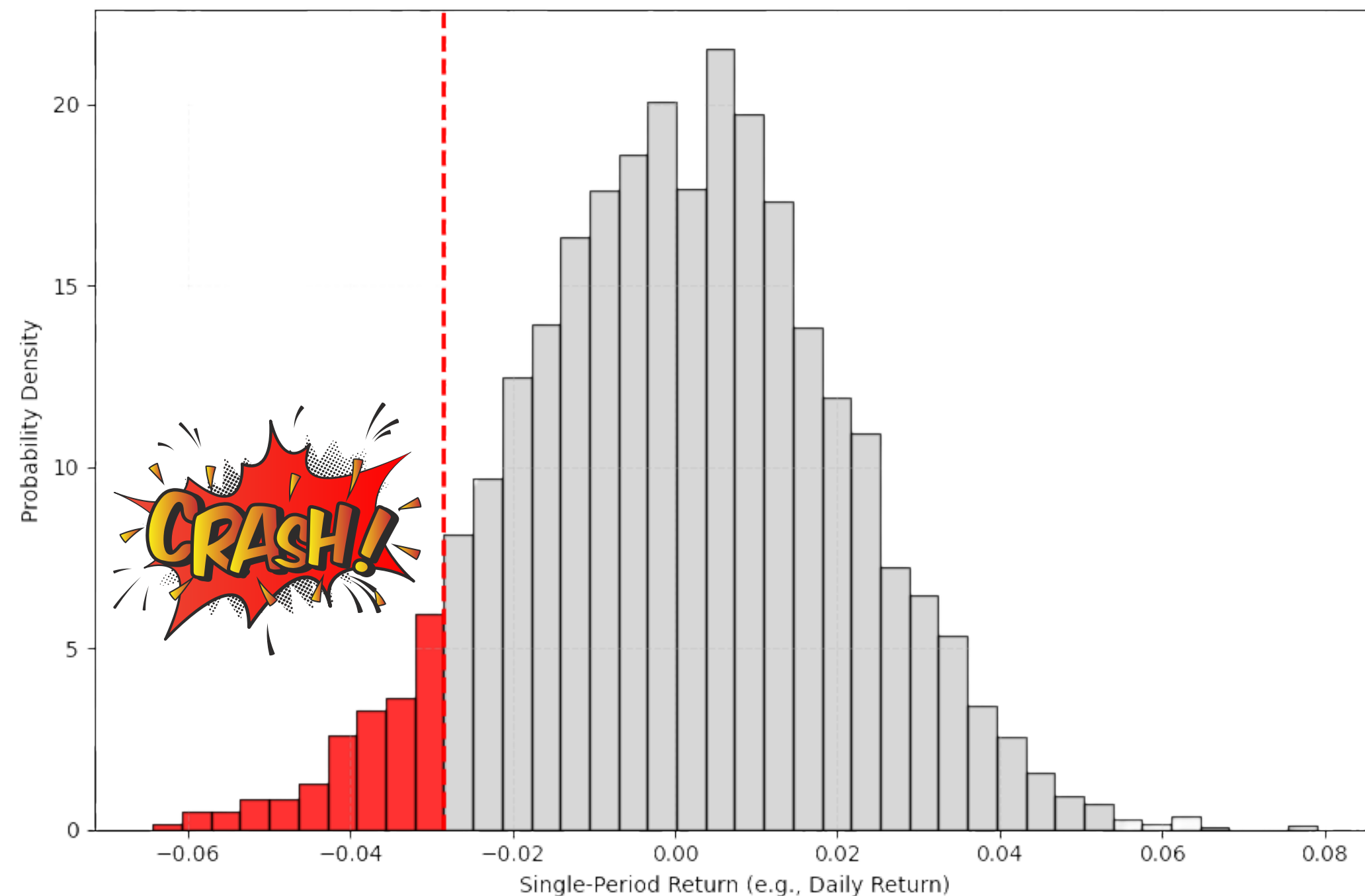
- 1** Overview of Key Findings
- 2** Data and Research Methodology
- 3** Key Findings and Mechanisms
- 4** Replicating the Data Part 1
- 5** Replicating the Data Part 2
- 6** Implications of the Study



PART 1: OVERVIEW OF KEY FINDINGS



Left-tail risk is the probability that an asset experiences extreme negative returns over a single time period (i.e. a crash).



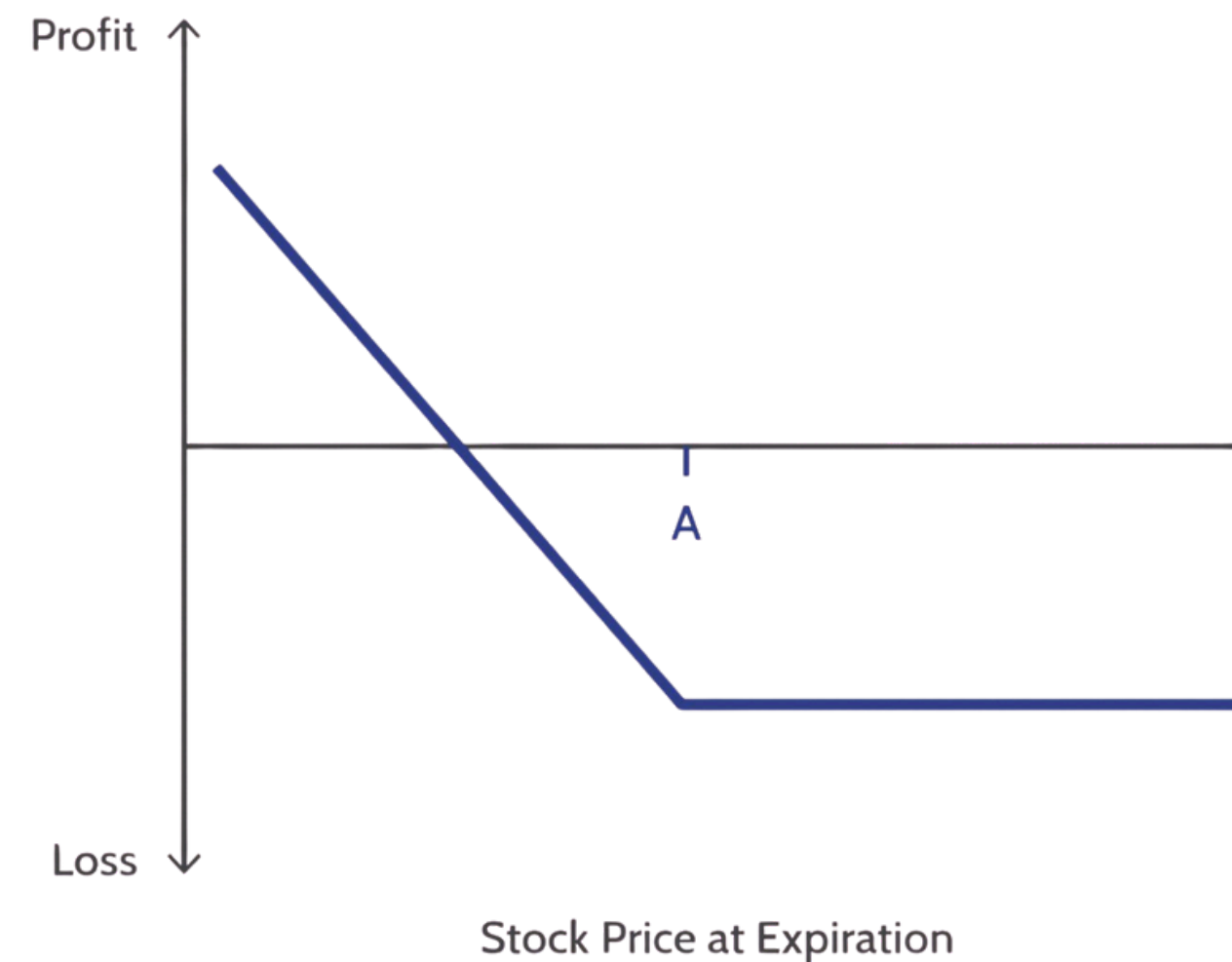
What Is Left-Tail Risk?



Crash insurance is insurance against crashes. It involves purchasing assets that safeguards against crashes.

What Is Crash Insurance?

**Crash insurance can come in many forms...
For example, buying a put option**



THE VALUE OF CRASH INSURANCE FOR ASSETS WITH HIGHER LEFT TAIL RISK SHOULD BE HIGHER

Just like car and health insurance...



Car Insurance

Drivers who have a **higher risk of getting into an accident** should pay a **higher premium**.

As such, on balance, they will **have similar returns on insurance** vis-a-vis safer drivers who pay a lower premium due to a smaller probability of getting into similar accident.



Health Insurance

Individuals with a **higher risk of developing severe illnesses** should pay a **higher premium**.

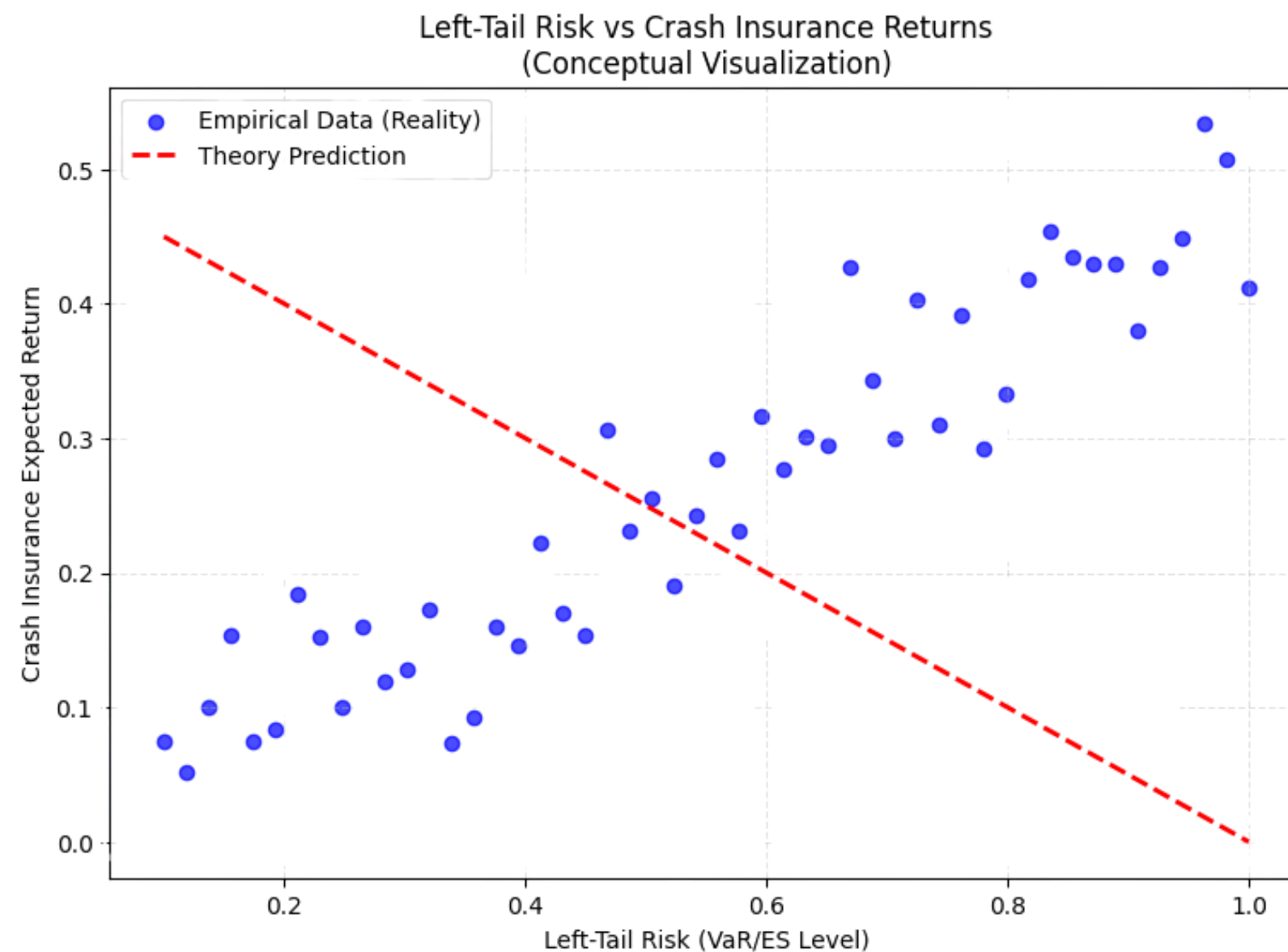
As such, on balance, they will **have similar returns on insurance** vis-a-vis healthier individuals who pay a lower premium due to a smaller probability of experiencing similar medical condition.

THIS SHOULD ALSO BE THE CASE FOR THE OPTIONS MARKET FOR CRASH INSURANCE ON ASSETS

“If options markets efficiently incorporate downside risks, firms with higher left-tail risk should have more expensive crash insurance.”

“Correspondingly, given that they are more expensive, the returns on crash insurance should on balance be similar for firms with different left-tail risks.”

HOWEVER, THE AUTHORS FOUND THAT THIS IS NOT THE CASE IN REALITY



“We find a significantly positive relation between firms’ left-tail risk and returns on crash insurance.”

IN SIMPLER TERMS, THIS MEANS THAT THE RETURNS ON CRASH INSURANCE WAS HIGHER FOR HIGH RISK ASSETS

This means that...



Reckless drivers were paying underpriced car insurance!

AND



Unhealthy individuals were paying underpriced health insurance!

***HOW DID THE AUTHORS
DETERMINE THIS?***



PART 2: **DATA AND** **RESEARCH** **METHODOLOGY**

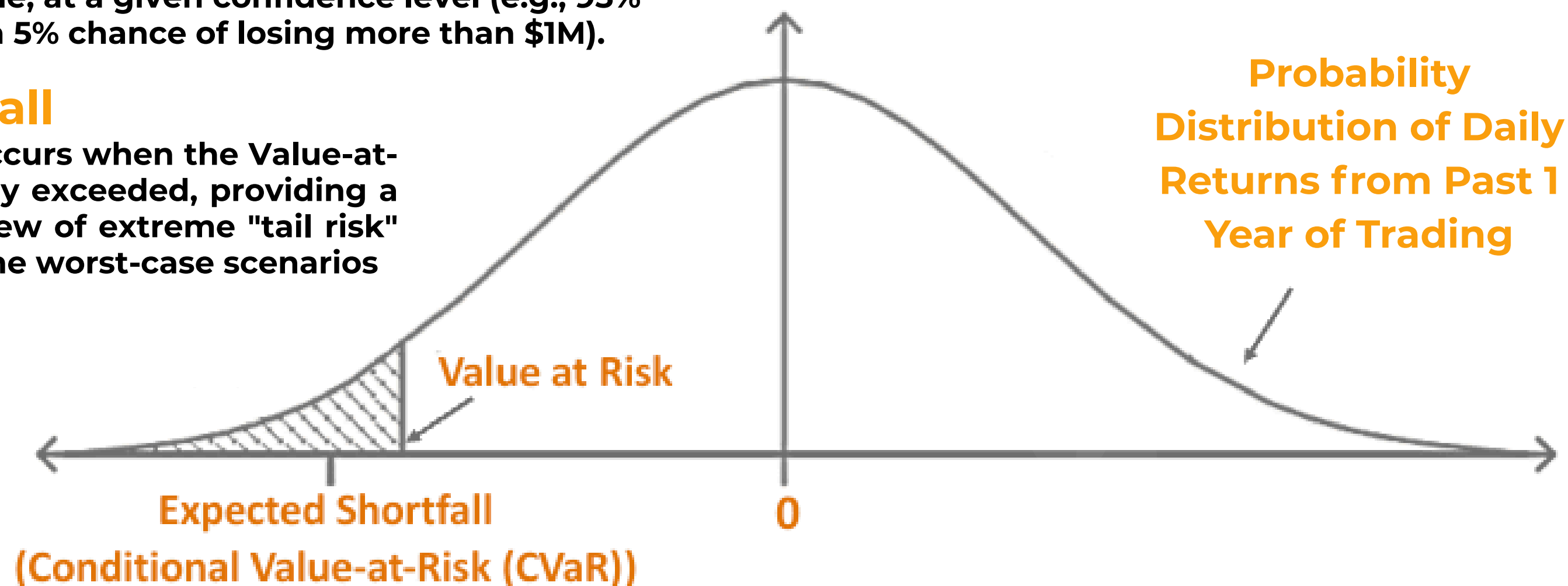
THE AUTHORS USED VALUE-AT-RISK AND EXPECTED SHORTFALL TO MEASURE THE ASSETS' LEFT-TAIL RISK

Value-at-Risk

A measure of the maximum potential loss on an investment over a specific time frame, at a given confidence level (e.g., 95% VaR of \$1M means only a 5% chance of losing more than \$1M).

Expected Shortfall

The average loss that occurs when the Value-at-Risk threshold is actually exceeded, providing a more comprehensive view of extreme "tail risk" by averaging losses in the worst-case scenarios



THEY ALSO USED A BEAR PUT SPREAD TO MEASURE THE CRASH INSURANCE RETURNS



**Bear Put
Spread**

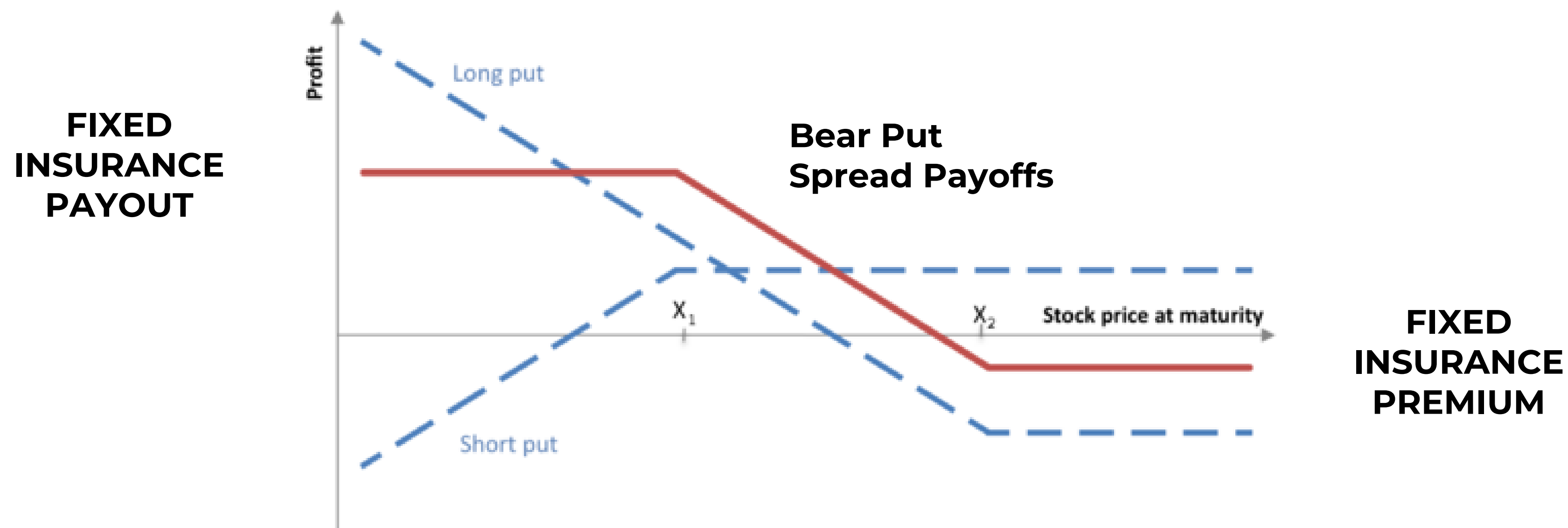
=

**Long
Put**

+

**Short
Put**

THIS GAVE A FIXED INSURANCE PREMIUM IN GOOD TIMES AND FIXED INSURANCE PAYOUT IN BAD TIMES



THE AUTHORS ALSO CONSTRUCTED 19 CONTROL VARIABLES FOR THE STUDY

Firm characteristics	Trading Characteristics	Option-Based Controls	Systematic Factor Exposures
<ul style="list-style-type: none"> • Size • Book-to-market • Leverage 	<ul style="list-style-type: none"> • Momentum • Short-term reversal • Illiquidity • Idiosyncratic volatility • Skewness • Kurtosis 	<ul style="list-style-type: none"> • Variance risk premium • Volatility-of-volatility • Risk-neutral skewness • Option demand 	<ul style="list-style-type: none"> • β BEAR • β STRAD • β JUMP • β VOL • β TAIL • β DOWN

THE AUTHORS USED ASSET LEVEL DATA FROM JAN 1996 TO DEC 2017 WITH 155,003 DATA POINTS



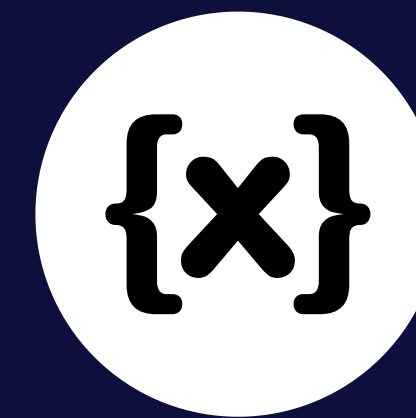
Used CRSP data for Stock Prices

Used in Calculation of Value-at-Risk and Expected Shortfall Variables and Trading Controls



Used OptionMetrics Data for Options Prices

Used in Calculation of Bear Put Spread Returns and Option-Based Controls



Used Compustat Data for Control Variables

Including information on Firm-Related Characteristics which Served as Controls

Caveat: Only Stocks with Exchange-Listed Options were Used

THE STUDY USED FOUR DIFFERENT METHODS TO DETERMINE THE RELATIONSHIP BETWEEN LEFT-TAIL RISK AND CRASH INSURANCE RETURNS

- 1 Univariate Analysis**
Sort firms into deciles based on left-tail risk and compare bear spread returns between highest versus lowest groups
- 2 Bivariate Analysis**
Same as univariate analysis, but sorting firms into deciles within each control group
- 3 Fama-Macbeth Regression**
Multivariate regression of bear spread returns on assets' left-tail risks
- 4 Systematic Risk Tests**
Multivariate regression of bear put spread returns on systematic risk control variables



PART 3: **KEY FINDINGS AND MECHANISMS**

THE STUDY FOUND THAT ASSETS WITH HIGHER LEFT TAIL RISK HAD HIGHER BEAR PUT SPREAD RETURNS

Methodology	Findings
Univariate Analysis	Assets in the top decile of left-tail risk had ~0.2% higher bear put spread returns than assets in the lowest decile of left-tail risk
Bivariate Analysis	Assets in the top decile of left-tail risk had significantly higher bear put spread returns than assets in the lowest decile of left-tail risk
Fama-Macbeth Regression	Assets with an additional 1% of left-tail risk had ~16-20% higher bear put spread returns
Systematic Risk Tests	Differences in bear put spread returns across assets with different levels of left-tail risk could not be explained by systematic market risk factors

The Puzzle:

**Why are Crash Insurance Returns Higher for
Assets with Higher Left-Tail-Risk?**

MECHANISM #1: THE MARKET UNDERREACTS TO PRICE VOLATILITY

When volatility begins to increase,



The market underestimates future increases in volatility...



As such, crash insurance is not valued sufficiently by the market to hedge against future volatility increases.

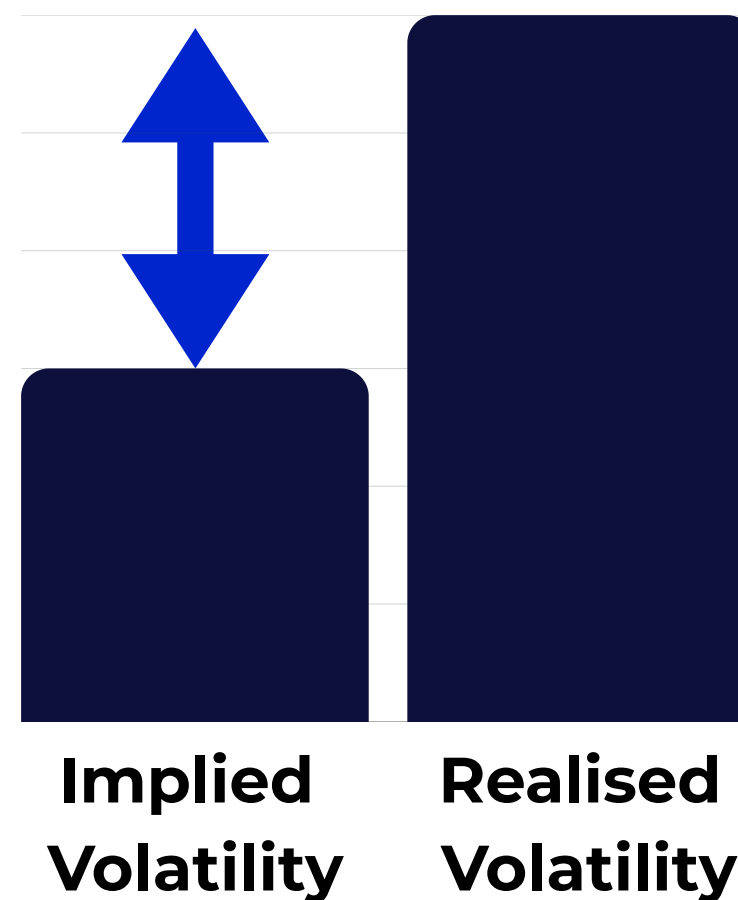
When volatility spikes later, the value of the crash insurance appreciates disproportionately.

THE AUTHORS DETERMINED THIS BY COMPARING THE IMPLIED VOLATILITY OF THE BEAR PUT SPREAD RETURNS AT PURCHASE WITH THE REALISED VOLATILITY

At Time of Purchase of Bear Put Spreads



At Maturity of Bear Put Spreads



- Firms with higher left-tail risk faced higher realised volatility at maturity
- However, the implied volatility at the time of purchase of the bear put spreads did not reflect the likelihood of volatility increasing more for firms with higher left-tail risk
- As such, the authors' hypothesised that **part of the reason for undervaluation of bear spreads was underexpectation of volatility spiking**

MECHANISM #2: THE MARKET UNDERESTIMATES THE PERSISTENCE OF LEFT-TAIL RISK

in other words, investors are buying the dip TOO SOON



FIRMS WITH HIGH LEFT-TAIL RISK OFTEN SEE “BAD DAYS” CLUSTERING TOGETHER

- Firms with high left-tail risk often see days with falling asset prices clustering together
- Explained simply, when the stock price crashes, they often continue to crash
- If the asset price falls significantly today, it is likely to continue over the next few days



THIS SHOULD RESULT IN THE VALUE OF CRASH INSURANCE INCREASING AFTER A “BIG DIP”

in other words...

“When the asset price dips, don’t buy the dip straightaway! Buy crash insurance first because the asset is going to dip further!!”

HOWEVER, RETURNS ON CRASH INSURANCE IN FACT INCREASED AFTER THE ASSET PRICE FELL



This means investors were “undervaluing” their crash insurance for future crashes.



Implies that investors were underreacting to the persistence of left-tail risk.



PART 4: **REPLICATING** **THE DATA** **PART 1**

WHILE THE PAPER IS GENERALLY ROBUST, IT IS IMPORTANT TO UNDERSTAND ITS ASSUMPTIONS

Factor	Assumption	Potential Limitations
Overall Dataset	<ul style="list-style-type: none"> Usage of US Data from 1996 to 2017 is a good reflection of reality Usage of Asset-Level Analysis is a good reflection of portfolios 	<ul style="list-style-type: none"> Unclear whether results are generalisable to other regions, timeframes and portfolios
Measure of Left-Tail Risk	<ul style="list-style-type: none"> Value-at-Risk and Expected Shortfall are good measures of left-tail risk Usage of Past 1 Year of Daily Returns is a good measure of left-tail risk 	<ul style="list-style-type: none"> Unclear whether results are generalisable to other forms of construction of left-tail risk
Measure of Crash Insurance	<ul style="list-style-type: none"> Bear Put Spreads are a good measure of crash insurance Usage of Options with 1 Month to Maturity are a good measure of crash insurance returns Usage of Midpoint of Bid-Ask spread provides a sufficient counterfactual Usage of Delta-Hedged Strike Prices reflect appropriate crash insurance prices 	<ul style="list-style-type: none"> Unclear whether results are generalisable to other forms of crash insurance
Measure of Control Variables	<ul style="list-style-type: none"> Firm Characteristics, Firms Characteristics, Option Based Controls and Systematic Market Risk Exposure Factors are sufficient controls 	<ul style="list-style-type: none"> Unclear whether there are other potential confounding factors which were not accounted for

WE FOCUSED ON DIVING INTO TWO LIMITATIONS WHEN SEEKING TO REPLICATE THE STUDY

Limitation

Approach

1

The study analysed crash insurance at the asset level

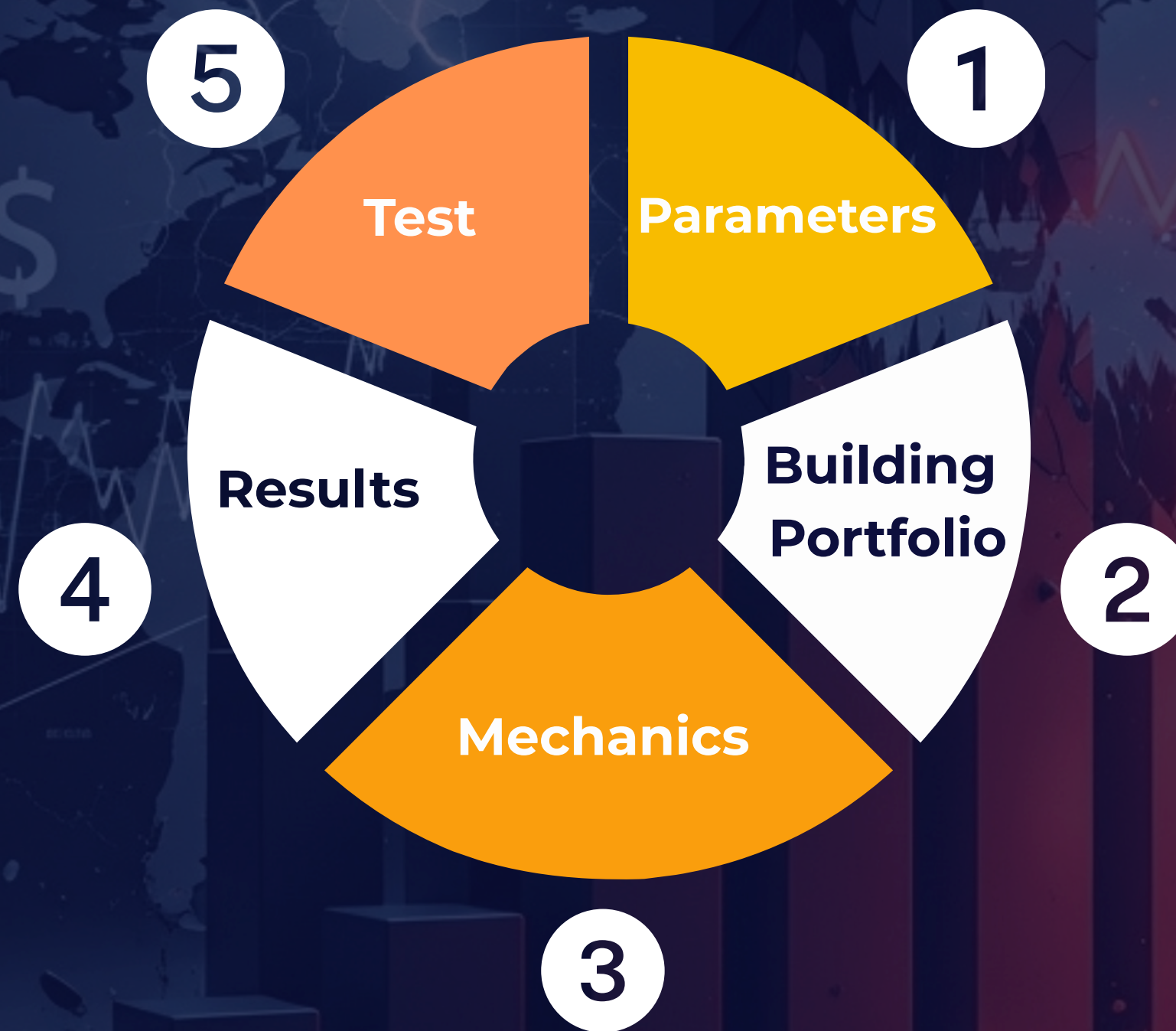
#1: To study crash insurance at the portfolio level

2

The study analysed crash insurance using bear put spreads

#2: To study crash insurance using simple long put options

OVERVIEW OF OUR SIMPLE CRASH INSURANCE PORTFOLIO METHODOLOGY



1 PARAMETERS

Simulation Timeline: 2 Jan 2015 - 19 Nov 2025

Crash Risk: 10%

Rebalancing Frequency: Quarterly

Hedge Ratio: 30%

Target Strike Price: 85% of current SPY (For Deep OTM put options)

Option Maturity: 1 Month

Rolling Schedule: New OTM puts are long monthly and held to expiry

Data Used: S&P 500 Historical Data, SPX options features data public and Volatility Risk Premium Data

2 BUILDING PORTFOLIO

Crash insurance = Long S&P 500 stock + Systematic option hedge that pays off during crashes

Equal-Weighted Hedge Portfolio

**Constant 30% Hedge Ratio
regardless of VIX**

Quarterly Rebalancing

Insurance cost is incurred monthly for options purchased, and payoff from expiring options is added.

Volatility-Weighted Hedge Portfolio

**Dynamic Hedge Ratio based on VIX:
50% if VIX is above median, otherwise 30%**

Quarterly Rebalancing

Insurance cost is incurred monthly for options purchased, and payoff from expiring options is added.

3 MECHANICS

Every month long new OTM puts with target strike price 85% of current SPY with 1 month expiry is bought to ensure continuity.

At expiry:

If $SPY > Strike$, options expire worth lesser. This would be deemed as insurance cost

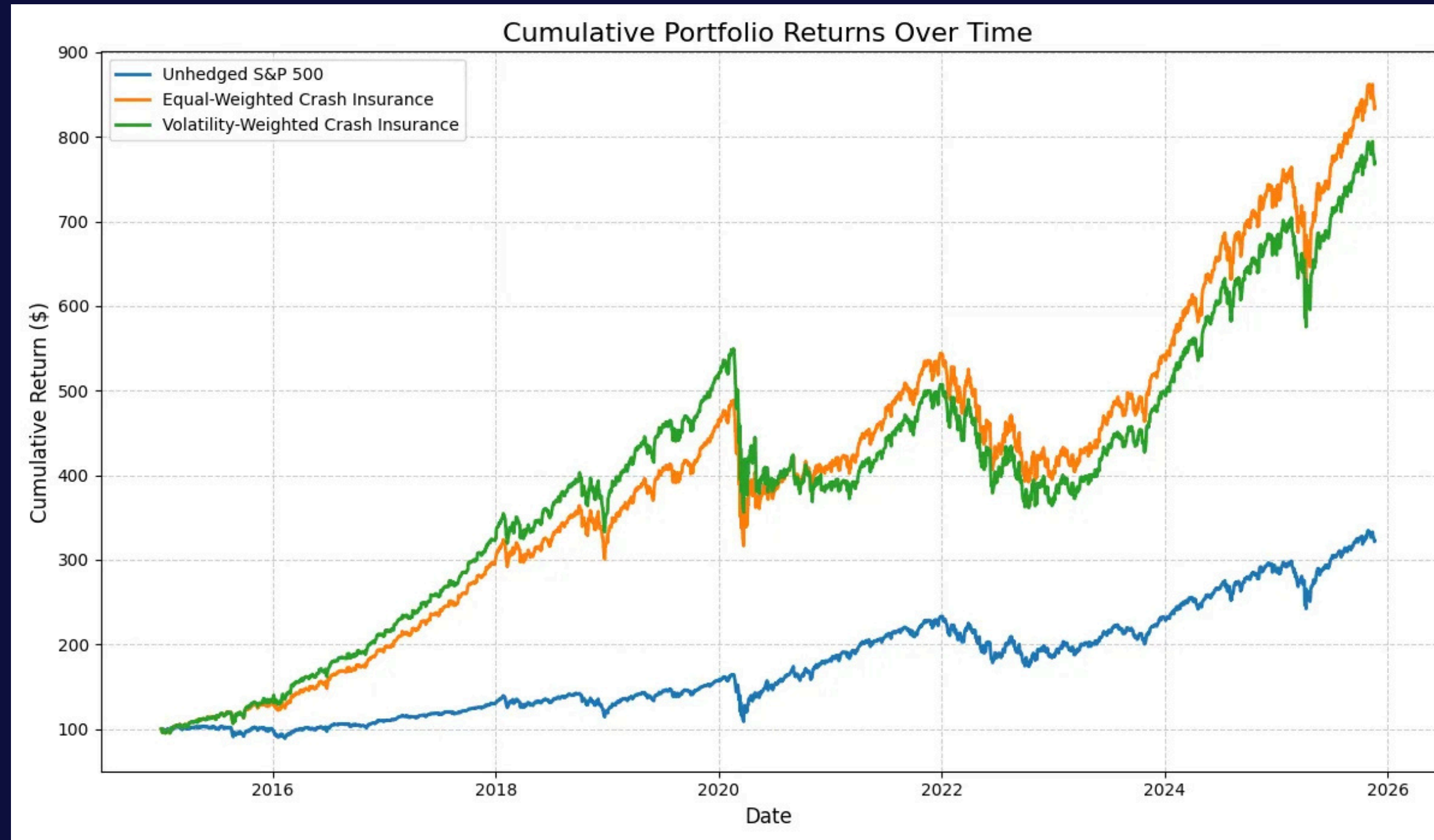
If $SPY < Strike$, options pay intrinsic value. This would be gained as crash protection

* Payoff = $\text{Max}(0, K - ST) \times 100 \times \text{Contracts}$ (Where K = Put Strike, ST = SPY price at expiry)

Final Portfolio Return = Portfolio Return - Option Cost + Crash Payoff

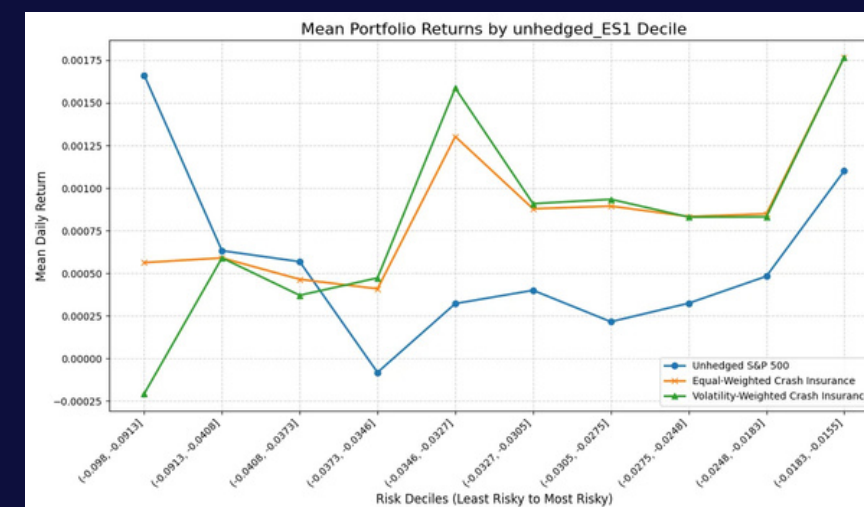
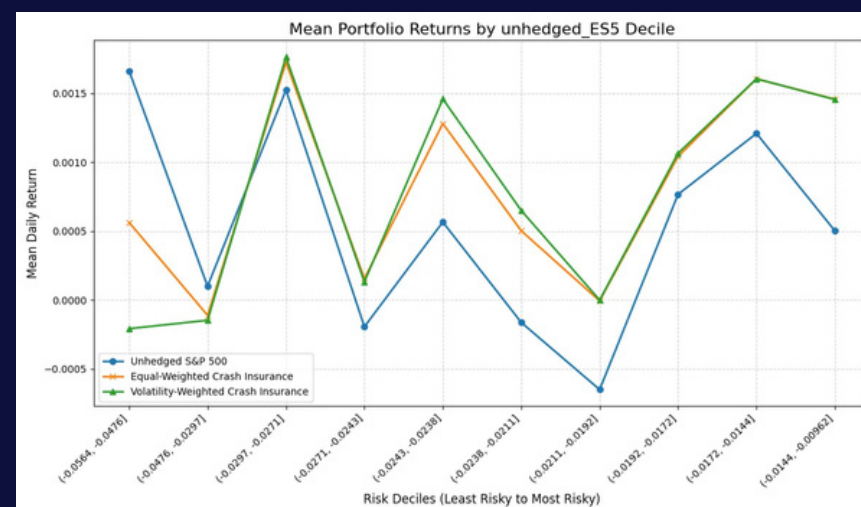
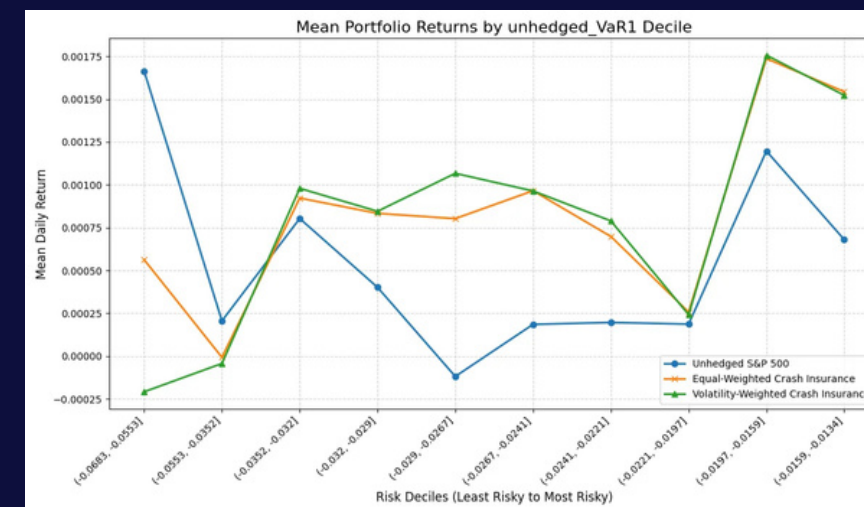
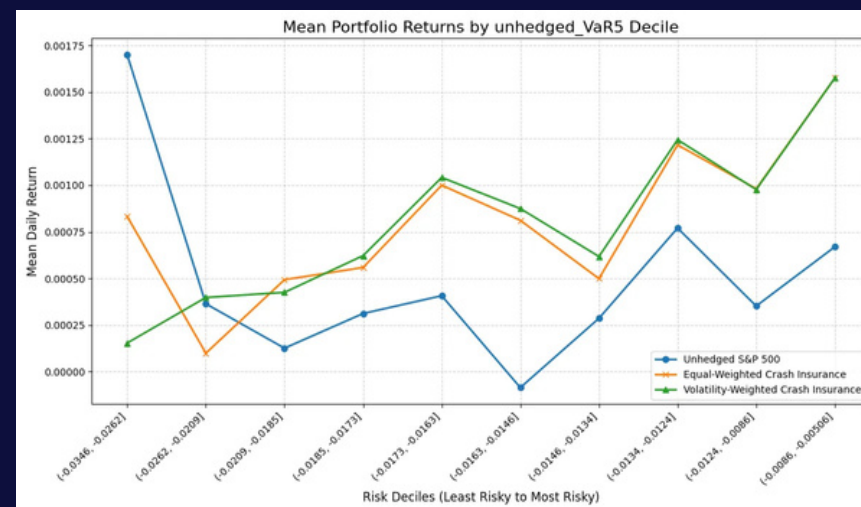
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RESULTS



5 TEST

- As risk level increases, in general there is an increase in returns due to returns becoming relatively more stable or show a less severe decline, demonstrating the protective benefit of the crash insurance.





PART 5: **REPLICATING** **THE DATA** **PART 2**

OUR SECOND APPROACH FOCUSES ON REPLICATING THE STUDY WITH ANOTHER CRASH INSURANCE MEASURE

Limitation

1

The study analysed crash insurance at the asset level

2

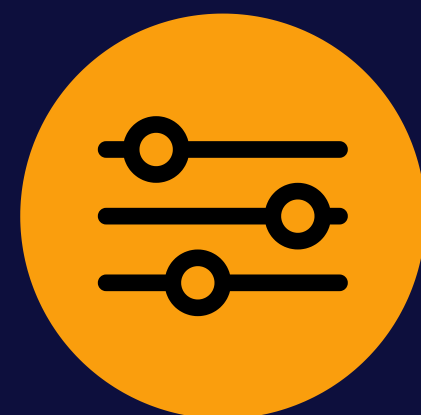
The study analysed crash insurance using bear put spreads

Approach

#1: To study crash insurance at the portfolio level

#2: To study crash insurance using simple long put options

FOR THIS, WE USED US OPTION LEVEL AND DAILY STOCK SECURITY DATA



US Option Level Output Data

By Wharton
Research Data
Services



Daily Stock Security Data

By **Centre for
Research in Security
Prices**

THE KEY CHALLENGE HERE WAS THE OVERWHELMING AMOUNT OF DATA VIZ. TIME FOR ANALYSIS

1

Too Much Data

Over 12 million data points from 1 year worth of data

2

Too Little Time

Challenging to calculate bear put spread returns

HENCE, WE FOCUSED ON A SMALL TIME FRAME AND USED ONLY LONG PUT OPTIONS

	Approach #1	Approach #2
Description	Focused only on options expiring on 20 Nov 2020	Focused only on Long Put Options as Crash Insurance
Rationale	<ul style="list-style-type: none">• Crash insurance would be highly valued given that it was at the height of the COVID-19 pandemic - most likely to “disprove” study• Left tail risk could be best estimated given that it was one year into pandemic	<ul style="list-style-type: none">• Most obvious and accessible form of crash insurance, particularly among retail investors• Hence, least likely to be “undervalued”, as opposed to bear put spreads, where “undervaluation” could be due to accessibility

FROM THERE, WE CONSTRUCTED OUR INDEPENDENT AND DEPENDENT VARIABLES

1

Dependent Variable

Return on long put option that expires on 20 Nov 2020, from 12 Oct 2020 to 30 Oct 2020

2

Independent Variables

95% Value-at-Risk and Expected Shortfall based on stock price from 14 Oct 2019 to 14 Oct 2020

THIS GAVE US 2,421 DATA POINTS BASED ON THE SAME NUMBER OF ASSETS

US Option Level Output Data in 2020 - 12.9 Million Data Points

US Option Level Output Data Expiring on 20 Nov 2020 - 599 Thousand Data Points

US Option Level Output Data with Available Security Data - 8,405 Data Points

US Option Level Output Data with Available Security Data on 12 Oct 2020 and 30 Oct 2020 - 2,421 Data Points

Notes:

- We did not use the delta hedge methodology to compute the appropriate strike price, but used all the strike prices available

FROM THERE, WE RAN TWO SIMILAR TESTS AS PER THE STUDY

1

Univariate Analysis

Comparison of top
decile with bottom
decile

2

Fama-Macbeth Regression

Regression of returns
on left-tail-risk
measures

FINDING #1: RETURNS ON LONG PUT OPTIONS WERE WORSE FOR FIRMS WITH HIGHER LEFT TAIL RISK

- To be precise, returns on long put options for firms in the top decile of left tail risk were about 200% lower than returns for firms in the bottom decile of left tail risk
- This runs contrary from the study

```
=== Test 1: Sorted by VaR_0.05 (descending) ===
```

```
Average return (top decile):      -2.475267
```

```
Average return (bottom decile): -0.274783
```

```
Difference (top - bottom):        -2.200485
```

```
=== Test 2: Sorted by ES_0.05 (descending) ===
```

```
Average return (top decile):      -2.982624
```

```
Average return (bottom decile): -0.494484
```

```
Difference (top - bottom):        -2.488140
```

FINDING #2A: FIRMS WITH HIGHER VALUE AT RISK HAD LOWER RETURNS ON LONG PUT OPTIONS

- Value-at-risk and returns on long put options were found to be negatively correlated
- In particular, 1% higher value-at-risk found to be correlated with 74% lower returns on long put options
- While not statistically significant, this also contradicts the study

```

=== Regression 1: Returns on VaR_0.05 ===
                        OLS Regression Results
=====
Dep. Variable:          returns    R-squared:                0.000
Model:                  OLS        Adj. R-squared:             -0.000
Method:                 Least Squares    F-statistic:            0.8226
Date:                   Mon, 01 Dec 2025    Prob (F-statistic):      0.365
Time:                   21:36:57          Log-Likelihood:         -14239.
No. Observations:       2416             AIC:                   2.848e+04
Df Residuals:           2414             BIC:                   2.849e+04
Df Model:                1
Covariance Type:        nonrobust
=====
                        coef      std err          t      P>|t|      [0.025      0.975]
-----
const                -1.5590         5.269     -0.296     0.767     -11.891      8.773
VaR_0.05             -74.8289        82.506     -0.907     0.365    -236.618     86.960
=====
Omnibus:              3847.223    Durbin-Watson:           1.950
Prob(Omnibus):         0.000    Jarque-Bera (JB):       2654927.514
Skew:                  10.022    Prob(JB):               0.00
Kurtosis:              164.157    Cond. No.               46.4
=====

```

FINDING #2B: FIRMS WITH HIGHER EXPECTED SHORTFALL HAD LOWER RETURNS ON LONG PUT OPTIONS

- Expected shortfall and returns on long put options were found to be negatively correlated
- In particular, 1% higher expected shortfall found to be correlated with 51% lower returns on long put options
- While not statistically significant, this also contradicts the study

```

=== Regression 2: Returns on ES_0.05 ===
                        OLS Regression Results
=====
Dep. Variable:          returns    R-squared:                0.000
Model:                  OLS        Adj. R-squared:             0.000
Method:                 Least Squares    F-statistic:              1.012
Date:                  Mon, 01 Dec 2025    Prob (F-statistic):       0.314
Time:                  21:36:57          Log-Likelihood:           -14239.
No. Observations:      2416             AIC:                     2.848e+04
Df Residuals:          2414             BIC:                     2.849e+04
Df Model:               1
Covariance Type:       nonrobust
=====
                    coef    std err          t      P>|t|      [0.025    0.975]
-----
const             -2.1583      5.370      -0.402      0.688     -12.688      8.371
ES_0.05           -51.8665     51.548     -1.006      0.314    -152.949     49.216
=====
Omnibus:            3845.909    Durbin-Watson:           1.950
Prob(Omnibus):       0.000    Jarque-Bera (JB):        2651471.525
Skew:                10.015    Prob(JB):                 0.00
Kurtosis:            164.053    Cond. No.                 29.1
=====

```


WHILE NOT STATISTICALLY SIGNIFICANT, THIS RAISES QUESTIONS ON THE STUDY'S FINDINGS

Conclusions

The paper's findings, while robust, may not hold in all situations

This may especially be when crash insurance is more easily valued

Limitations

Results are not statistically significant, and did not include controls

Dataset is small and limited to a single period only

Areas to Study

Alternative forms of crash insurance - such as long put options

Differences in crash insurance value across different market contexts

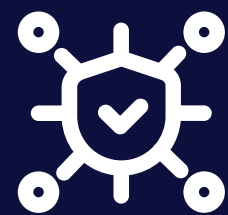


PART 6: IMPLICATIONS OF THE STUDY

DRAWING THINGS TOGETHER: USING CAR INSURANCE AS AN ANALOGY



**Left-Tail Risk =
Accident/Catastrophic Risk**



**Crash Insurance =
Catastrophic Accident Policy**



**Crash Insurance Returns =
Profitability of the Policy**



**A car fell into a sinkhole
when it formed on
Tanjong Katong Road South
on July 26.**



EXPECTATION VS ANOMALY

	Expected Scenario (Fair Pricing)	Anomaly (Underreaction)
Car Insurance Analogy	Drivers with the highest accident risk should pay the highest premiums and they should expect negative returns (i.e., it costs money to maintain the protection).	Drivers with the highest accident risk can buy underpriced policies such that, over time, they earn positive returns.
Supporting Basis	Our findings using real data from 2020.	Our findings using simulated data.

IMPLICATIONS OF THE PAPER'S FINDINGS FOR BUSINESS OWNERS



Owners should not trust calm markets and must proactively act before market realizes the risk



Persistence of left-tail risk requires proactive risk management



Use crash insurance proactively as it is most valuable before market realizes the risk

Implication #1:
Owners should
not trust calm
markets and must
act proactively



As options markets underreact to rising crash risk, businesses' true exposure to left-tail risk is higher than it seems.

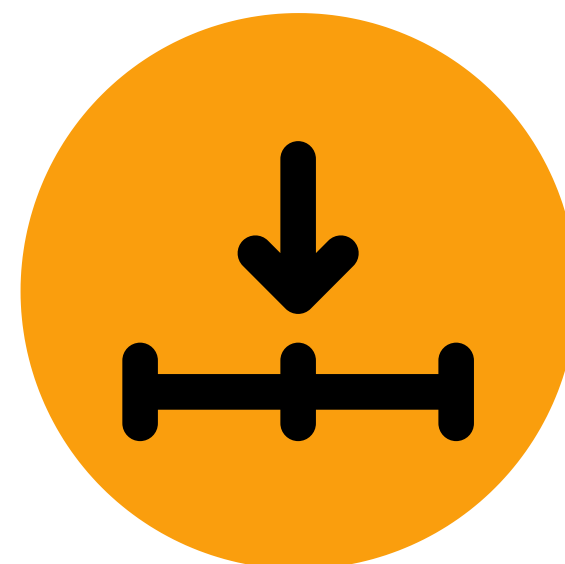


For firms with high left-tail risks, any failure to obtain crash insurance is costly as protection is underpriced.

Implication #2: **Persistence of** **Left-Tail Risk** **requires proactive** **management**



77% of the firms in the highest left-tail risk portfolio (Decile 10) remain in Deciles 9 and 10 a year later



Owners must treat small signs as early warnings; e.g. Stock price near 52-week low, rising volatility, declining margins

Implication #3:
**Use crash
insurance
proactively when
it is most valuable
before markets
realize the risk**

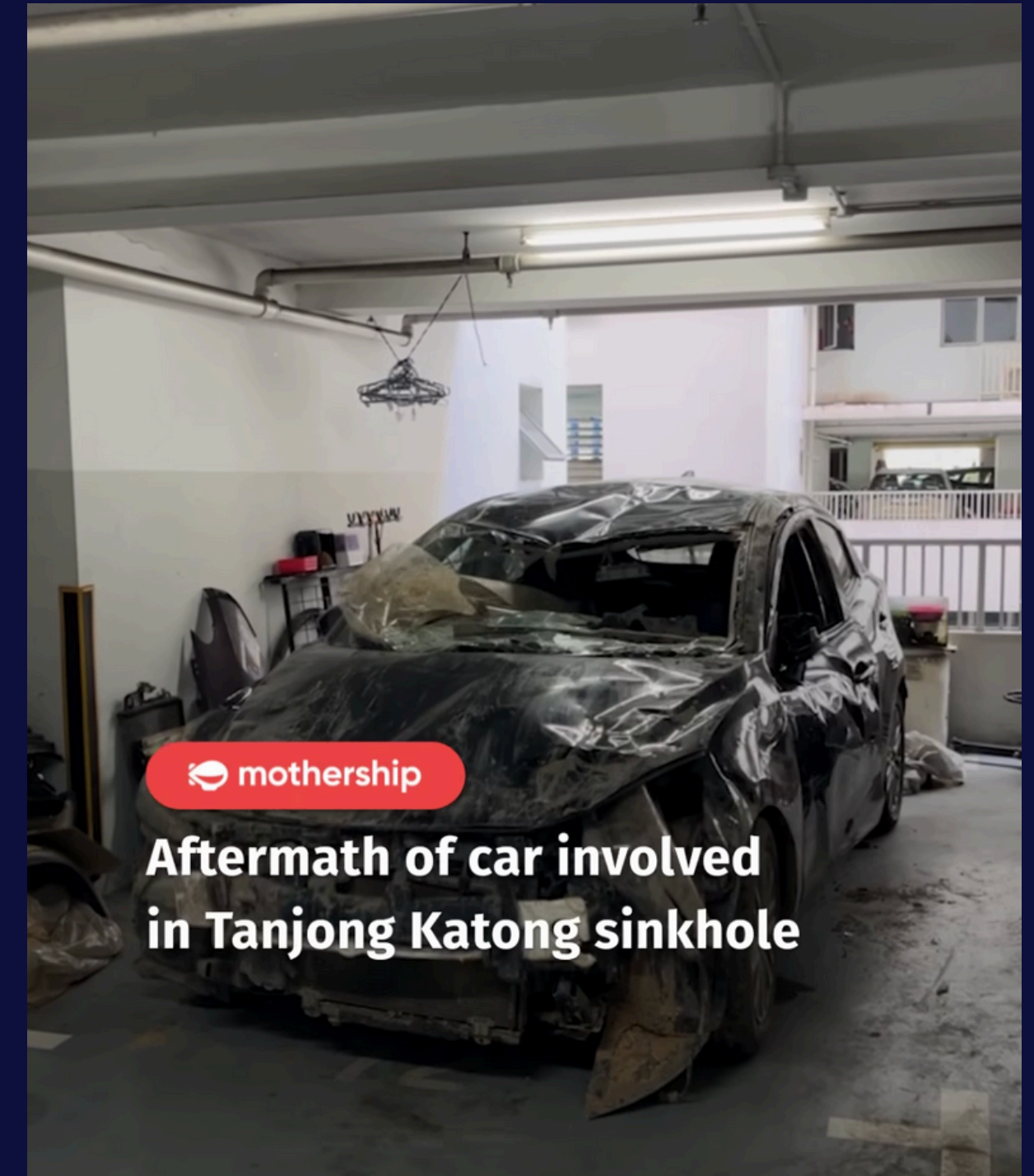


Crash insurance is most profitable when when investor sentiment suggests markets are overlooking future risk



Crash insurance can be an offensive strategic tool instead of a defensive tactic

HOW CAN WE GUARD AGAINST “SINK-HOLE” EVENTS?



LEFT-TAIL RISK ASSESSMENT FRAMEWORK

PHASE 1



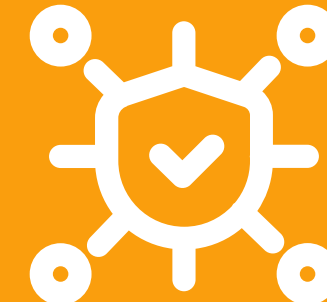
Perform Left-Tail Risk Stress Testing

PHASE 2



Identify Signs of Market Underreaction

PHASE 3



Choose Mitigation/Hedge/ Transfer Strategies

PHASE 4



Monitoring “Tail Persistence”

PHASE 1

PERFORM LEFT-TAIL RISK STRESS TESTING



Operational Tail Risk

E.g., Supply chain volatility, regulatory exposure



Financial Tail Risk

E.g., Leverage ratios, liquidity buffers



Market-Based Signals

E.g., Industry stock volatility, competitor tail events

Actions Include:

- Running worst-case cash-flow scenarios
- Stress-test liquidity, supply chain, and debt structure
- Evaluate customer concentration and operational single points of failure
- Model substantial revenue drop (e.g., 20%-50%) to assess the business's resilience

PHASE 2

IDENTIFY SIGNS OF MARKET UNDERREACTION



Is the business in a state where negative info is slow to be priced in?



Has the business's risk increased without external visibility?



Are external conditions masking the business's internal fragility?

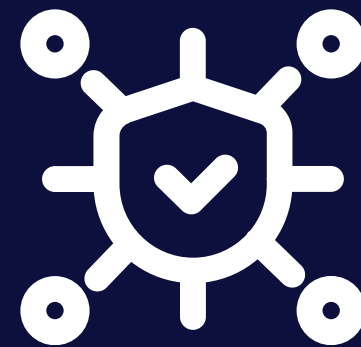
PHASE 3

CHOOSE STRATEGIES



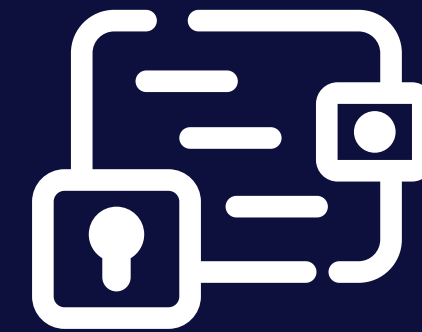
MITIGATE

internally if the left-tail
risk arises from the
business model



HEDGE

using underpriced crash
insurance to protect
against left-tail risk

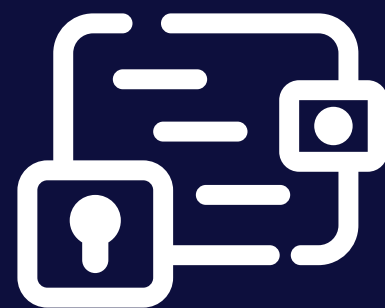


TRANSFER

externally to shift the
left-tail risk to another
party

PHASE 4

MONITOR “TAIL PERSISTENCE



Use dashboards
to monitor
metrics monthly

- **Internal indicators:** Cash burn rates, customer concentration, short-term debt proportion?
- **External indicators:** Industry volatility, credit spreads, commodity/FX swings?



Establish a crash severity index
and automatic response triggers
when tail scores moves up index

Score	Meaning
1	Mild downside risk
2	Manageable but noticeable downside risk
3	Elevated asymmetric downside
4	High chance of extreme outcomes
5	High left-tail risk with systemic amplification

THANK YOU!

BEAR PUT SPREAD RETURNS WERE BASED ON PUT OPTIONS WITH ONE MONTH TO MATURITY

- Bear put spreads were constructed based on put options with a one month maturity
- Returns were calculated based on the value of the bear put spread at maturity

$$\text{RETURN} = \frac{(\Delta_{2,t} - \Delta_{1,t})S_{t+1} + \max(K_1 - S_{t+1}, 0) - \max(K_2 - S_{t+1}, 0)}{(\Delta_{2,t} - \Delta_{1,t})S_t + P_1 - P_2} - 1$$

BONUS: ADDITIONAL TECHNICAL DETAILS IN CALCULATION OF BEAR PUT SPREADS



Issue #1: What is the appropriate strike price for the long and short puts?

Solution: Use delta hedging - based on 30% and 10% chance of finishing in the money respectively



Issue #2: How do we know what the options could be sold for in reality?

Solution: Use the midpoint of the bid and ask prices to determine the prices which the options could be bought or sold